

# Inhomogeneity in doped Mott insulator

Dung-Hai Lee<sup>1,2,3</sup>

(1) *Department of Physics, University of California, Berkeley, CA 94720, USA*

(2) *Material Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

(3) *Center for Advanced Study, Tsinghua University, Beijing 100084, China*

*We introduce the concept that there are two generic classes of Mott insulators in nature. They are distinguished by their responses to weak doping. Doped charges form cluster (i.e. distribute inhomogeneously) in type I Mott insulators while distribute homogeneously in type II Mott insulators. We present our opinion on the role inhomogeneity plays in the cuprates.*

## 1. INTRODUCTION

Understanding of Mott insulators is one of the central problems of condensed matter physics. In addition to electronic Mott insulators recently an interesting Bose Mott insulator has been produced by trapping bosonic neutral atoms in an optical lattice.<sup>1</sup> The recent upsurge in interest in Mott insulators is largely due to their dramatic response to doping. For example by doping the transition metal oxides such as the cuprates and the manganites, one obtains superconductors with the highest known transition temperatures and the world's most magnetoresistive materials.

Mott states, in addition to being insulating, can be characterized by the presence or absence of a spontaneously broken symmetry (e.g. spin antiferromagnetism), by the nature of the low energy excitation spectrum (e.g. gapped or gapless),<sup>2</sup> and most recently, by the presence or absence of topological order and charge fractionalization.<sup>3</sup> To this list, we add a "type" index that classifies Mott insulators into two types depending on their response to doping.

In a type I Mott insulator an increasing chemical potential induces a first order phase transition from an undoped state to a charge-rich state

D.-H. Lee

so that the charge density changes discontinuously. In other words, there is a range of “forbidden charge density,” and two-phase coexistence. In a type II Mott insulator, charges go in continuously above a critical chemical potential. Depending on the ratio between the delocalization energy and the interaction energy these doped charges can form a Wigner crystal or a homogeneous liquid phase.<sup>4</sup> An important implication of this work is the existence of a generic class of Mott insulators which become inhomogeneous upon doping.

For the transition metal oxides discussed above, there exists considerable evidence that light doping induces spatial inhomogeneity.<sup>5,6,7,8,9</sup> Conversely, for Mott insulator such as  $Sr_{1-x}La_xTiO_3$ ,<sup>10</sup> conventional Fermi liquid behavior is observed for doping as low as  $x = 5\%$ , indicative of homogeneity. Thus, the former are type I, the latter is type II.

Generally speaking, insulating states with charge gap including both band<sup>11</sup> and Mott insulators occur in crystalline systems at isolated rational “occupation numbers,”  $\nu = \nu^*$ , where  $\nu$  is the number of particles per chemical unit cell. By “doping” we mean a process which causes the occupation number to shift away from  $\nu^*$ . When lattice translation symmetry is not spontaneously broken,  $\nu^*$  is typically an integer for bosons, and an even integer for fermions with spin. The fermionic state thus may be adiabatically connected to a weakly interacting band-insulator, however a Bose Mott insulator is always a strong correlation effect. Charge gapped insulating states can also occur when  $\nu^*$  is a rational fraction (for fermions this includes odd-integer). Usually when that happens, translational symmetry is spontaneously broken so that the unit cell of the reduced translation group has integral  $\nu$  for bosons<sup>12,13</sup> and even integral  $\nu$  for fermions. For instance, electronic Mott insulators with  $\nu^* = 1$  often exhibit antiferromagnetic Néel long-range order, which doubles the unit cell leading to an effective  $\nu_{eff} = 2$ . Nevertheless there exists model bosonic systems for which the Mott state can be shown to have no broken symmetries for  $\nu^* = 1/2$ <sup>15,14,3</sup>. (Currently, no laboratory system has been found which unambiguously exhibits this exotic behavior.)

The rest of this paper addresses the response of Mott insulators to light doping (i.e.  $\nu \rightarrow \nu^* - \epsilon$ ). Our central observation is that regardless of their classification (i.e. symmetry, gap, topology) Mott insulators can be divided into two groups differentiated by whether they remain homogeneous after doping.<sup>16</sup> Furthermore we shall show that these two types of insulating state is analogous to the two types of superconducting state under magnetic field. In particular if the Mott insulators are two dimensional and the constituent particles are bosons there exists a mathematical mapping, the so-called “duality transformation”, that relates their zero-temperature

### Inhomogeneity in doped Mott insulator

response to doping to the finite-temperature response of a 3D superconductor to a magnetic field.<sup>12,18</sup> (The duality transformation has been used to deduce two types of doping behavior by Balents *et al* in the context of doping a spin liquid called “nodal liquid”<sup>19</sup>) The following table summarize correspondence between the two.

<i>T = 0 properties of 2D Bose Mott insulators</i>	<i>T &gt; 0 properties of 3D Superconductors</i>
Doping	Applying magnetic field
Chemical potential $\mu$	Applied magnetic field $H$
Induced particle density $\rho$	Magnetic induction $B$
World line of doped particles	Flux tubes
Quantum delocalization of doped particles	Thermal meandering of flux tubes
Type I Mott insulator	Type I superconductor
Mott gap	$H_c$
Effective attraction between doped particles	Positive N-S interface energy
Type II Mott insulator	Type II superconductor
Effective repulsion between doped particles	Negative N-S interface energy
Mott gap	$H_{c1}$
Wigner crystal of doped particles	Abrikosov flux lattice
Superfluid state	Entangled vortex fluid
Critical $\mu$ at which Wigner crystal melts	$H_{c2}$

## 2. DOPING A TRANSLATIONALLY INVARIANT BOSE MOTT INSULATOR

In order to avoid the issue of spontaneous symmetry breaking in electronic Mott insulators and focus on the absolute essentials, we consider the simplest kind of Mott insulators - the ones formed by spin zero point bosons on a lattice. Due to Ref.<sup>1</sup> this consideration is no longer an academic exercise.<sup>20</sup> Consider the following Hamiltonian

$$\begin{aligned}
 H = & -\frac{t}{2} \sum_{\langle ij \rangle} (a_i^\dagger a_j + h.c.) + \frac{U}{2} \sum_i (a_i^\dagger a_i)(a_i^\dagger a_i - 1) \\
 & + \frac{1}{2} \sum_{i,j} V_{ij} a_i^\dagger a_i a_j^\dagger a_j - \mu \sum_i a_i^\dagger a_i,
 \end{aligned} \tag{1}$$

where  $i, j$  label the lattice sites on a D-dimensional hypercubic lattice, and  $a_i^\dagger$  creates a boson at site  $i$ . The first term of Eq. (1) describes the quantum mechanical “hopping” of bosons from a site  $i$  to its nearest neighbors  $j$ , the second and the third terms describe the pair-wise interaction between bosons. The  $U$  term is a contact interaction and the  $V_{ij}$  terms describe interaction between bosons separated by  $|\mathbf{r}_i - \mathbf{r}_j|$ .

### D.-H. Lee

Let us focus on the limit where  $U/t \rightarrow \infty$ . In this “hardcore” limit doubly occupied site costs energy  $U$ , hence are excluded. In this case for large positive  $\mu$  there is a unique ground state

$$|\text{Mott}\rangle = \prod_j a_j^\dagger |0\rangle, \quad (2)$$

in which each site is occupied by one and only one boson and hence  $\nu = 1$ . Since this state is separated from all other states with the same particle number by an energy gap of order  $U$ , clearly we have an insulator.

The behavior of the system upon decreasing  $\mu$  (i.e. doping) depends on the values of  $V_{ij}$ . For  $V_{ij} = 0$  the doped holes (i.e. the empty sites) interact only through the hardcore exclusion. For appropriate  $\mu$  where the ground state has a small  $1 - \nu$ , the hopping term is clearly minimized by delocalizing the holes around, since there is plenty of room available for them. In fact it is known that such system is a uniform superfluid with superfluid stiffness proportional to  $t(1 - \nu) \ln \ln[1/(1 - \nu)]$ .<sup>17</sup> Conversely, if  $V_{ij}$  is attractive (negative) with range  $|\mathbf{r}_i - \mathbf{r}_j| \leq R$  it is clear that the holes will cluster when the strength of this interaction is strong compared with  $t$  - the holes will phase separate. Thus depending on the value of  $V_{ij}$  Eq. (1) can describe either a type I or type II Mott insulator when  $\nu = 1$ .

Consider, for example, the case in which  $V_{ij} = -V$  for  $(ij)$  nearest-neighbor sites, and  $V_{ij} = 0$  otherwise. Technically in this limit Eq. (1) is equivalent to the  $S=1/2$  ferromagnetic XXZ model in a  $z$ -direction magnetic field

$$H = -J_{xy} \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - J_z \sum_{\langle ij \rangle} S_i^z S_j^z - h_z \sum_i S_i^z. \quad (3)$$

The mapping between these two models relates  $J_{xy}$  to  $t$ ,  $J_z$  to  $V$ ,  $h_z$  to  $\mu + Vc/2$  ( $c$  = the coordination number). The  $z$  component of the magnetization is related to the boson density according to  $M_z = (\nu - 1/2)N$ . (Here  $N$  is the total number of lattice sites.) For  $J_z < J_{xy}$  (i.e.  $V < t$ ), this model has XY order in the absence of  $h_z$ . In this range of parameters varying  $h_z$  causes the magnetization  $M_z$  to vary continuously. Thus for each fixed  $M_z$  the ground state is uniform and ferromagnetic. In terms of bosons this means that the doped system is a uniform superfluid for all  $\nu$ . Conversely, for  $J_z > J_{xy}$  (i.e.  $V > t$ ) the model is effectively an Ising ferromagnet with fully polarized ground state. In this case  $M_z$  exhibits a discontinuity  $\Delta M_z = N$  at  $h_z = 0$ . Thus for all  $|M_z| \neq 1$  the ground state exhibits phase separation into two oppositely polarized domains. In terms of the bosons, these two domains are Mott insulating ( $\nu = 1$ ) and empty ( $\nu = 0$ ) respectively.

## Inhomogeneity in doped Mott insulator

### 3. DUALITY TRANSFORMATION FOR THE BOSE MOTT INSULATOR IN D=2

While the above discussions applies to arbitrary space dimensions, there is a particularly convenient way of thinking about Bose Mott insulators that is specific to  $D = 2$ . It turns out that for the class of model given by Eq. (1), there is a mathematical mapping, the “duality” transformation, that provides us an alternative view of the physics of Eq. (1) in terms of the vortices of the boson field. It is this mapping that enables us to establish a precise connection between the two types of Mott insulators with type I and type II superconductors.<sup>18,12,19</sup> In the following we discuss the physical content of the duality transformation without going into its technical details of the transformation.<sup>12</sup>

A vortex is a topological defect in the Bose field. When a boson is adiabatically transported around a vortex, the boson wavefunction acquires a phase factor - the Aharonov-Bohm phase. In the dual picture, the vortices are viewed as particles (they turn out to have Bose statistics as well), and when they are brought around an original boson they acquire an Aharonov-Bohm phase. The fact that a boson and a vortex acquire a phase when they go around one another implies that bosons and vortices can not Bose condense simultaneously. As is well known, in the Bose superfluid phase the vortex density fluctuation must be absent. Conversely, in the dual phase, where the vortices form a superfluid, the boson density (and hence the dual magnetic flux) must be frozen; the vortex superfluid phase is the Mott insulating phase of the original bosons.<sup>21</sup> It is important to note that the absence of boson density fluctuation is a necessary but not sufficient condition for vortex condensation. For example a *static* boson density can still frustrate vortex condensation because it acts like a background magnetic field. However when the static boson density corresponds to an integral  $\nu$ , the vortices see a background magnetic flux corresponding to integral number of flux quanta per plaquette. (The vortices live on the dual lattice, i.e., the centers of the square plaquettes.) This type of flux is “invisible” to the vortices because they can be “gauged away”. The  $\nu = 1$  Bose Mott insulator, discussed in the previous section, corresponds to precisely this situation.

When the boson density is a fraction ( $\nu = p/q$ ) it is also possible for the vortices to condense. Such a state is most naturally accompanied by spontaneous translation symmetry breaking, as discussed above, leading to an enlarged unit cell with an effective integer  $\nu$ .<sup>12</sup> However, it is possible to imagine a more exotic Mott state at  $\nu = p/q$  in which the translation symmetry is unbroken. This could happen if  $q$  elementary vortices form a bound-state, and these composites then condense. Such a composite condensation is unfrustrated by an uniform static boson density  $\nu = p/q$ .<sup>3</sup> Moreover,

**D.-H. Lee**

since the condensate vortex consists of  $q$  elementary vortices, the charge  $1/q$  bosonic soliton excitation (viewed by the condensate vortices as a flux quantum) can become a finite energy excitation. Thus there is fractional charge solitons!

In short, a vortex condensate requires the bosons to Mott insulate. Consequently we can view a boson Mott insulator as a vortex superconductor. Doping changes the average background boson density. To the vortices this appears as a change in the background magnetic field. In this way the doping properties of the boson Mott insulator is related to the magnetic properties of the vortex superconductor.

#### **4. THE ANALOGY WITH SUPERCONDUCTORS IN A MAGNETIC FIELD**

In the above discussion a zero temperature boson Mott insulator is mapped onto a zero-temperature vortex superconductor (with quantum fluctuating two-dimensional electromagnetic fields).<sup>22</sup> The final step is to realize that the quantum partition function of a (particle-hole symmetric) two dimensional superconductor with fluctuating gauge field is equivalent to the classical (i.e. thermal) partition function of a three-dimensional superconductor with thermally fluctuating magnetic field.<sup>23</sup> This final correspondence between the Mott insulator and the classical fluctuating 3D superconductor is summarized in table I.

A lot is known about thermally fluctuating 3D superconductors. Mean-field theory predicted that there are two types. For a type I superconductor the magnetic induction  $B$  jumps discontinuously from  $B = 0$  to  $B = H$  at  $H = H_c$ . As the result there is a range of  $B$  (i.e.  $0 < B < H_c$ ) in which phase separation occurs. For example one way of enforcing a fixed average magnetic induction is to place a flat slab of type I superconductor under magnetic field  $H < H_c$ . It is known that when that is done “intermediate state” where superconducting region and normal region alternate occurs. It is interesting that among many possible inhomogeneous structures the laminar structure (or stripe) has been observed.<sup>24</sup> Translate this using table I we conclude that a type I Bose Mott insulator undergoes a first order insulator  $\rightarrow$  superfluid transition as a function of chemical potential. The density of doped bosons jumps discontinuously at the transition. When the doping density is fixed at a value smaller than the critical density at the first order transition the system phase separates.

Mean-field theory predicts that in a type II superconductor the magnetic induction  $B$  increases from zero continuously at the lower critical field  $H_{c1}$ . For  $H > H_{c1}$  the magnetic induction appear in the form of flux tubes each

### Inhomogeneity in doped Mott insulator

enclosing a single quantum of magnetic flux. These flux tubes form a regular lattice in the absence of disorder. Unlike the type I superconductors thermal meandering of the flux tubes can affect the physics of type II superconductors dramatically near  $H_{c1}$  and  $H_{c2}$ . For example near  $H_{c1}$  the distance between neighboring flux tubes is much greater than the range of their interaction (the London penetration depth  $\lambda$ ). As the result thermal meandering of the flux tubes can melt the flux lattice into flux liquid.<sup>25</sup> At larger magnetic field the density of flux tube becomes higher so that their interaction can stabilize the flux lattice. This flux lattice persists until  $H \rightarrow H_{c2}$  where the thermal meandering melt the flux lattice again. Translating the above using table I implies that the extra carriers enter a type II Mott insulator continuously at a critical chemical potential. Once they enter they can either delocalize (hence Bose condense) or form a Wigner crystal depending on the relative importance of the delocalization energy and the interaction energy. At at very low carrier density the delocalization always win and we expect the boson hopping to render the system a superfluid.

What determines a superconductor to be type I or type II is the ratio between the London penetration depth and the core size of the vortices, or more physically in terms of the sign of the interface energy between the normal and superconducting regions. Type I superconductors have a positive interface energy while type II superconductors have a negative one. As the result the flux tubes effectively attract each other in type I superconductors while repel each other in type II superconductors. As we have seen this is exactly how we turn a type I Mott insulator into a type II one in Eq. (1).

## 5. DOPING A BOSE MOTT INSULATOR WITH AN ORDER PARAMETER

As we discussed at the beginning of this paper a Mott insulating state can be accompanied by translation symmetry breaking. For example let us consider Eq. (1) with  $U/t \rightarrow \infty$  and  $V_{ij} = +V$  for nearest neighbor  $\langle ij \rangle$  and 0 otherwise. For sufficiently strong  $V$  and  $\mu = 0$  the ground state breaks translation symmetry and bosons form a checkerboard lattice and Mott insulate. In this two-fold degenerate ground state the unit cell is doubled. Is this Mott insulator type I or type II?

With the above specific choice of  $U$  and  $V_{ij}$  Eq. (1) is equivalent to Eq. (3) with  $J_{xy} = -t$ ,  $J_z = V$  and  $h = \mu - Vc/2$ . For this choice of parameters it is known that as a function of  $h_z$  Eq. (3) exhibits a “spin flop” transition from the antiferromagnetic Ising ( $S_z$ ) ordered phase into the ferromagnetic XY ordered phase. Translate this into the boson language it implies that the boson Mott insulator is type I.

D.-H. Lee

Before closing this section we argue that the existence of an order parameter ( $\psi$ ) in the insulating state (for the above example  $\psi$  is the two-sublattice density wave order parameter) has an effect in determining the type of the Mott insulator. If we assume that doping changes the value of  $\psi$ , then at the interface between doped and undoped region a spatially varying  $\psi$  is necessary. Through the spatial gradient energy

$$\int d^d x |\nabla \psi|^2 \quad (4)$$

a positive contribution to the surface energy is resulted. Consequently the presence of an order parameter in the insulating state drives the system toward a type I Mott insulator.

## 6. What ROLE DOES INHOMOGENEITY PLAY IN THE CUPRATES; PAIRING IN THE HOLE-RICH ISLANDS IN

$$Bi_2Sr_2CaCu_2O_{8+x}$$

Finally we come to address the question *what role does inhomogeneity play in the cuprates?* One of the most direct evidence of electronic inhomogeneity in the cuprates comes from the STM image of the surface of  $Bi_2Sr_2CaCu_2O_{8+x}$ , where nanoscale spatial variation of the energy gap in the tunnelling density of states is observed.<sup>6,7</sup> (Of course we have to assume that the STM results is not a surface artifact.) As emphasized by Lang *et al.*,<sup>7</sup> the characteristics of the tunnelling spectra divide into two distinct types: “ $\alpha$  region” where sharp coherence peaks exist ( $\Delta$  ranges from 25 -50 meV), and “ $\beta$ ” region where there is no sharp coherence peak ( $\Delta$  ranges from 50 to 75 meV). We take these evidences as suggesting that in the  $\alpha$  region superconducting pairing has clearly won over all other competing orders, while in the  $\beta$  region it has not. We believe that inhomogeneity allows the superconducting  $\alpha$  region to exist in the underdoped regime. As the result superconductivity prevails over a wider range of doping concentration. However, we do not think inhomogeneity plays an important role in the pairing of  $\alpha$  regions. The above line of thinking suggests that in order to understand the pairing mechanism it is best to concentrate on the  $\alpha$  regions.

Since the superconducting gap can vary over the length scale of a few nanometers, the coherence length must not be significantly longer than that. At such length scale (comparable with the averaged inter-hole distance) the Coulomb interaction is poorly screened. How can pairing tolerate such strong unscreened Coulomb interaction?

In the rest of this section we present an explicit example of a paired state that can survive strong repulsion. The state we shall concentrate on is



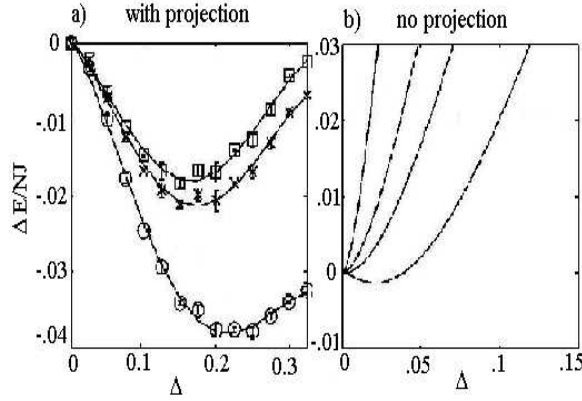


Figure 1

Fig. 1. Results for  $x = 12\%$ .  $\Delta E$  as a function of  $\Delta$  from Gutzwiller projection in a  $10 \times 10$  lattice. Open circles: pure t-J model; crosses: t-J model with nearest neighbor repulsion  $V_{nn} = 3J$ ; squares: t-J model with Coulomb interaction  $V_c = 3J$ .

of the form

$$|\psi(\Delta)\rangle = P(n_i)|\psi_{dBCS}(\Delta)\rangle. \quad (5)$$

In Eq. (5)  $|\psi_{dBCS}(\Delta)\rangle$  is a d-wave BCS state characterized by the gap parameter  $\Delta$ . The operator  $P$  is a Jastrow correlator that suppresses configurations with large density fluctuations. A particular form of  $P$ , that is often used in the high  $T_c$  context, is the Gutzwiller projection operator. In our opinion although we do not know precisely what Hamiltonian gives rise to a ground state like Eq. (5), it is rather safe to assume that the ground state in the  $\alpha$  region is of that form.

In the following we demonstrate that a state like Eq. (5) has the property that it is more sensitive to magnetic interactions than to charge interactions. To do that we consider the following Hamiltonian

$$\begin{aligned} H = & -t \sum_{\langle ij \rangle} (c_{j\alpha}^\dagger c_{i\alpha} + h.c.) + J \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) \\ & + V_c \sum_{i>j} \frac{1}{r_{ij}} (n_i - \bar{n})(n_j - \bar{n}). \end{aligned} \quad (6)$$

The purpose of the following discussions is not to prove Eq. (5) is the ground state of Eq. (6). Rather we hope to demonstrate one point - the state given in Eq. (5) is more sensitive to the magnetic interaction  $J$  than to the charge interaction  $V_c$ . The result we shall quote below is obtained by Wang and collaborators in Ref.[26].

By a straightforward Monte-Carlo minimization Wang *et al* conclude that for doping  $x = 0.12$  it is energetically favorable to develop a non-zero

D.-H. Lee

$\Delta$  for  $V_c$  as big as  $9J$ . To appreciate the effects of the Jastrow correlator  $P$  (which they take as  $P = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})$  in Eq. (5) they compared the results with and without  $P$ . Interestingly without  $P$  a nearest-neighbor repulsion  $V_{nn}$  destabilizes pairing when  $V_{nn}$  is larger than  $\approx 0.5J$ ! Thus the Jastrow correlator  $P$  enhances the stability of pairing in Eq. (5) by as much as a factor of 18!

The following is a brief summary of Wang *et al*'s results. Given Eq. (5) Wang *et al* minimize  $E(\Delta) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$  by varying  $\Delta$ . The results presented below are obtained for  $x = 0.12$  in  $10 \times 10$  lattice by variational Monte-Carlo.

In Fig.1(a)  $\Delta E \equiv E(\Delta) - E(0)$  versus  $\Delta$  is plotted. The wavefunction used is Eq. (5) with  $P$  being the Gutzwiller projector. The open circles are for the pure t-J model (i.e. without charge-charge interaction), the crosses are for t-J model with a nearest neighbor repulsion  $V_{nn} = 3J$ , and the open squares are for t-J + Coulomb model (Eq. (6)) with  $V_c = 3J$ . For each of the three cases a nonzero  $\Delta$  develops.

In Fig.1(b)  $\Delta E \equiv E(\Delta) - E(0)$  versus  $\Delta$  is plotted for the repulsive nearest-neighbor ( $V_{nn}$ ) model. An important difference is that the Jastrow correlator  $P$  in Eq. (5) is *removed*. This time the optimal  $\Delta$  vanishes for  $V_{nn} \geq 0.5J$ .

These results clearly indicate that a strongly correlated paired state such as Eq. (5) can indeed sustain strong repulsion between the electrons. In addition we take this as suggesting a state like Eq. (5) is more sensitive to magnetic interactions than the charge interactions.

In the literature it is often stated that pairing correspond to real space binding of holes. In the presence of strong Coulomb interaction real space bound hole pairs are extremely energetically unfavorable, and indeed in an ansatz such as Eq. (5) no hole binding is present.

## 7. CONCLUSIONS

To summarize, in this paper we introduce the concept that there are two types of Mott insulator in nature. A type I Mott insulator becomes inhomogeneous after doping, while a type II Mott insulator remains homogeneous. We present a specific lattice boson models which Mott insulates without symmetry breaking at  $\nu = 1$  and, depending on the sign of a microscopic interaction, exhibits these two types of ground states after doping. We argue that the presence of a symmetry breaking order parameter in the insulating state has the effect of driving the system toward type I. In addition, we argue that these two types of Mott insulating states are dual to the type I and type II superconducting states. Here we conjecture that the

## Inhomogeneity in doped Mott insulator

notorious  $x = 0.19$  “quantum critical point” in the cuprates is related to the inhomogeneous to homogeneous transition when the magnetic induction ( $B$ ) in a type I superconductor is varied across  $H_c$ .

We also express our opinion on the role played by the inhomogeneity in the cuprates. To reiterate, we do not think inhomogeneity plays a central role in pairing. However it does allow the superconducting region (where pairing dominates over other competing orders) to protrude into the underdoped regime. As the result superconductivity prevails over a wider range of doping concentration. Finally we believe a correlated pairing state given by Eq. (5) can describe the hole-rich superconducting regions (the so-called  $\alpha$  regions) imaged by the STM. Using the result of Ref.[26] we demonstrate that this state is much more sensitive to spin-spin interaction than charge-charge interaction. This provides a mechanism by which pairing can survive poorly screened Coulomb interaction.

## ACKNOWLEDGMENTS

Part of this paper (two types of Mott insulator) is the result of a collaboration with S.A. Kivelson. I thank Seamus Davis and members of his group for valuable discussions. DHL is supported by NSF grant DMR 99-71503.

## REFERENCES

1. M. Greiner *et al*, Nature, **415**, 39 (2002).
2. A Mott insulator necessarily has a charge gap, however it is possible for it to have a gapless spin excitation spectrum.
3. T. Senthil and M.P.A. Fisher, Phys. Rev. B **63**, 134521 (2001); C. Nayak and K.Shtengel, Phys. Rev. B **64**, 064422 (2001); X-G Wen, cond-mat/0107071; L. Balents, M. P. A. Fisher, S. M. Girvin, cond-mat/0110005.
4. Presumably, in the absence of long-range forces, at low enough doping density, the charges in a type II Mott insulator always form a uniform fluid, while in the presence of long-range Coulomb forces, and at  $T = 0$ , they always form a Wigner crystal.
5. For a review of stripes in the cuprates see V. J. Emery, S. A. Kivelson, J. M. Tranquada Proc. Natl. Acad. Sci. USA **96**, 8814 (1999).
6. C. Howald, P. Fournier, and A. Kapitulnik, Phys. Rev. B **64**, 100504 (2001); S. H. Pan *et al*. Nature, **413**, 282 (2001).
7. K.M. Lang *et al*, Nature **415**, 412 (2002).
8. For a review of the possible inhomogeneity in the magnites see, e.g., E. Dagotto, T. Hotta, A. Moreo, Physics Report **344**, 1 (2001).
9. See, e.g., J.M. Tranquada, J. Phys. Chem. Solids, **59**, 2150 (1988).
10. Y. Tokura *et al*, Phys. Rev. Lett. **70**, 2126 (1993).
11. A band insulator insulates because of Pauli exclusion principle. In the non-interacting prototype of this type of insulator all available single-particle orbitals

## D.-H. Lee

are occupied. This type of insulator usually possesses a even number of electrons per crystalline unit cell.

12. D-H Lee and R. Shankar, Phys. Rev. Lett., **65**, 1490 (1990).
13. M. Oshikawa, Phys. Rev. Lett. **84**, 1535 (2000).
14. R. Moessner and S.L. Sondhi, Phys. Rev. Lett. **86**, 1881 (2001).
15. D.S. Rokhsar and S.A. Kivelson, Phys. Rev. Lett. **61**, 2376-9 (1988).
16. In the presence of long-range Coulomb forces macroscopic charge inhomogeneities is impossible as  $\nu - \nu^* \rightarrow 0$ . However, so long as the long-range forces are not too strong, there could exist a range of low doping in which the doped charges tend to cluster. We still refer to this kind of system as type I Mott insulators as well. For instance, if a low density of doped charges form puddles or stripes, we consider this a form of micro-phase-separation, while a Wigner crystal of doped charges is deemed homogeneous. This is because in a Wigner crystal charges localize in the smallest unit hence no clustering is exhibited.
17. D.S. Fisher and P.C. Hohenberg, Phys. Rev. B **37**, 4936 (1988).
18. M.P.A. Fisher and D-H Lee, Phys. Rev. B **39**, 2756 (1989).
19. L. Balents, M.P.A. Fisher and C. Nayak, Int. J. Mod. Phys. B **12**, 1033 (1998).
20. The system considered in this Ref.<sup>1</sup> can be “doped” by varying the period of the trap lattice.
21. We note that the vortex superfluid discussed here differs from the generic boson superfluid because the net vorticity is zero. The field theory that describes such “particle-hole symmetric” condensate has space and time on equal footing (i.e. relativistic). This fact makes the later analogy with thermal fluctuating 3D superconductors completely appropriate.
22. The field strength  $F_{\mu\nu}$  ( $\mu = 0, 1, 2$ ) of a 2+1-dimensional EM field has three independent components. The correspondence of these three components with the boson three-current  $(\rho, \mathbf{j})$  is as follows:  $F_{12} \leftrightarrow \rho, F_{01} \leftrightarrow j_2, F_{02} \leftrightarrow j_1$ .
23. In the correspondence between a quantum superconductor at  $T = 0$  and a thermally fluctuating superconductor at finite temperature the magnetic field ( $F_{12}$ ) and electric fields ( $F_{01}, F_{02}$ ) in the 2+1 dimensional quantum theory is mapped on to  $B_z$  and  $B_y, -B_x$  respectively.
24. See.e.g., M. Tinkham “Introduction to superconductivity”, McGraw-Hill, New York (1975); P.-G. de Gennes, “Superconductivity of metals and alloys”, Benjamin, New York (1966).
25. D.R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988).
26. Q-H Wang, J. H. Han and D-H Lee, Phys. Rev. B **65**, 054501 (2002).